

# Developing simulation and optimization approaches to support the planning of mental health care services: predicting demand and designing a network of services

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## Abstract

Mental health care problems currently represent one of the leading causes of disability and morbidity in many European countries. As a result, an increasing demand for mental health care is predicted for coming years across these countries. Still, the current supply of mental health services is far from being enough to satisfy this growing demand for care, and the current economic crisis can seriously hinder the development of such supply. Within this context, planning mental health care networks currently represents a health policy priority across European countries. This study aims to develop a multi-objective mathematical programming model - *MHC* model - to aid health planners in the management, design and planning of networks of mental health services within the context of NHS-based countries. The proposed model integrates estimates on future demand for mental health care based on the characteristics of the population with potential need of such care, predicted within the scope of this study. The applicability of the model is shown through its application to the mental health care sector in the Great Lisbon region in Portugal under different planning contexts.

**Keywords:** Mental health care, Estimates on future demand, Mathematical Programming Models, Network Planning

## 1. Introduction

Mental health is described by the World Health Organization (WHO) as “a state of well-being in which an individual realizes his or her abilities, can cope with the normal stresses of life, can work productively and is able to make a contribution to his or her community [1].

A variety of mental health care provision paradigms can be found across Europe. In particular, mental health care includes treatment, rehabilitation, and promotional and preventive activities, delivered in primary health care centers (PHCCs), general hospitals, psychiatric hospitals and through services in the community [2]. The most recent mental health policy developed by the WHO focuses on rehabilitation and social integration. For this purpose, the WHO recommends that countries should replace large psychiatric hospitals by community services through a deinstitutionalization process.

Over the past years, the provision of mental health care in Portugal has undergone restructur-

ing in order to transit from a hospital-based therapy to a model of continued and family-oriented. One of the measures taken to accomplish this was the implementation of a national plan developed in accordance to WHO recommendations. Persisting problems related to difficulties in accessing mental health care and the high prevalence of mental disorders are still being addressed. Nevertheless, great improvements in mental health care provision have already been achieved, namely, the development of community services, the setup of the basis for launching continuous care and the beginning of the deinstitutionalization process followed by the closure and restructuring of some psychiatric hospitals.

Nowadays, European countries face major demographic, social and economic changes that affect the well-being of the population and the provision of quality care [1]. The ageing phenomenon and the increase in the prevalence of mental disorders lead to an increase in the demand for mental health care. However, the current mental health care supply is

not enough to satisfy this growing demand for care.

Besides this, the current economic crisis imposes severe budget cuts along with a pressure to reduce public health care spending, which can seriously hinder the development of such supply. Consequently, there has been a worldwide awareness on this matter, leading to huge challenges and pressures concerning the development and improvement of mental health care provision in Portugal and many other European countries [1]. Within this context, a proper planning and use of mental health care resources is a health policy priority for these countries.

Mathematical programming models have been widely used to support health care planning in general and have potential to be used to assist policy makers and health planners in the organization of networks of mental health care. However, when it comes to consider the specificities of the mental health care sector, there is still little research. Moreover, most of the mathematical programming models have been developed to support the planning of mental health care services in the US context (examples can be found in Franz et al. [3] and Leff et al. [4]). Thus, there is evidence on the need to plan mental health care networks operating in the context of National Health Service (NHS) systems. Furthermore, studies constructing future demand estimates in order to be integrated into mathematical programming models to support planning of mental health care networks are being increasingly recognized in the literature.

In this line of thought, this study aims to predict the demand for mental health care services and develop a mathematical programming model - *MHC* model - to aid health care planners in the management, design and planning of networks of mental health care services, both at a strategical and tactical level, in the medium-term.

The proposed model provides information for planning, both in terms of services location, capacity planning and allocation of patients to services, while considering the specificities of the mental health care sector, namely: the multiple services - institutional care (IC), ambulatory care (AC), home-based care (HBC) and rehabilitation services: residential unit (RU) and occupational unit (OU)- and the multiple objectives relevant for planning in NHS-based systems.

The *MHC* model, implemented in the General Algebraic Modelling System (GAMS), is thus an innovative multi-objective mathematical programming model that gives support to health planners on how to re(organize) a multi-service network of mental health care through an adequate planning, in the context of NHS-based countries.

The paper is organized as follows: Section 2

provides a literature review of on existing methods to plan the delivery of mental health care services, including approaches to predict future demand for health care services and mathematical programming models for support health care planning in general, and mental health care in particular. Section 3 presents a description of the developed methodology, followed by the case study in Section 4 where the main results are presented and discussed. Finally, in Section 6 the main conclusions are drawn.

## 2. Literature Review

Mathematical programming models have been widely used in the health care planning literature. Within this area of research, existing models differ in several aspects, namely: in the planning purpose; in the number and types of services accounted for; in the number and types of objectives pursued; and in the consideration or not of uncertainty aspects.

### 2.1. Health care planning

#### Single and multi-service models

Most of existing models in the health care planning literature account for one single service (for instance Oliveira and Bevan [5]). The multi-service nature of health service delivery has been explored in the past years, but there is still little research on these. Mestre et al. [6] presents an example of a multiple service approach. The mathematical programming model developed aims to inform decision on the location and supply of hospital services, considering the multi-service structure of hospital production (including inpatient care, emergency care and external consultations). Also, Santibáñez et al. [7] presented a mathematical model to plan an inpatient hospital network, in particular the location selection of 34 clinical services (such as general medicine, ophthalmology and cardiac surgery) across hospitals and distribution of bed capacity.

#### Single and multi-objective models

The planning of health care services delivery usually depends on several conflicting objectives [8]. The management of these objectives is very challenging, and for that reason most of the existing studies on health care propose single-objective models [9]. Nevertheless, there has been an increasing interest in the development of multi-objectives approaches. Within the health care literature, equity, efficiency, costs and health gains are the most commonly used objectives.

Equity is a key objective pursued in NHS based-systems [10, 11], and for that reason is one of the objectives most widely used in the literature. However, there is no single definition of equity that can be used to plan the delivery of health care

services and there is little consensus on how equity should be measured [12]. Different equity concepts have been used, for instance equity of access [6], geographical equity [13], equity of utilization [5] and socioeconomic equity [14]. Cardoso et al. [13] explored the joint analysis of multiple equity objectives, by proposing a model to support planning decision in the long-term care sector that accounts for three equity-related objectives (equity of access, geographical equity and socioeconomic equity).

Efficiency and equity objectives have been frequently combined in the health care literature. An example is the bi-objective model proposed by Mitropoulos et al. [15] for locating hospitals and PHCCs considering (1) the minimization of the travel distance between patients and assigned facilities; and (2) an equitable distribution of facilities among citizens.

In the health care literature, cost minimization has also been commonly addressed. Syam and Côté [16] developed a model for the location and allocation of specialized health care services for the Department of Veterans Affairs (VA), including two criteria: (1) the VAs cost of providing service, including the fixed and variable treatment costs and lost patient cost (cost per patient not treated); and (2) the service rate provided to VAs patients, defined as the proportion of eligible patients served by the VA for a given geographical area.

Furthermore, the maximization of population's health was considered in the mathematical model proposed by Koyuncu and Erol [17] for optimal resource allocation decisions in countries where a risk of pandemic influenza may exist. These resources include monetary budget for antivirals and preventive vaccinations, Intensive Care Unit beds, ventilators and non-Intensive Care Unit beds. The model considers three objectives: (1) minimization of the number of deaths; (2) minimization of the number of cases; and (3) minimization of total morbidity days during a pandemic influenza.

To deal with multiple objectives, different approaches have been employed within the health context: (i) the multiple objectives can be written as a single objective function, using for example the weighting or goal programming methods; and (ii) a Pareto frontier can be built, by identifying compromise solutions (Pareto optimal solutions), which represent solutions that cannot be improved in one objective function without deteriorating their performance in at least one of the rest [18].

Cardoso et al. [13] uses the the weighting method to deal with the multiple equity objectives. The weights are built with preference information of the decision maker (DM), using the Measuring Attractiveness by a Category-Based Evaluation Technique (MACBETH). The goal programming

approach is used by Oddoye et al. [19] to allocate resources in a medical assessment unit. Mitropoulos et al. [15] uses the constraint method to derive the Pareto optimal solutions, between the conflicting objectives.

### Deterministic and stochastic models

The mathematical programming models can also be divided in deterministic and stochastic. Deterministic models are based on initial conditions and parameters with no uncertainty associated. Stochastic programming models arise when some of the data in the model are uncertain but can be specified by a probability distribution [20]. Within the health care area, the main studies developed consist in deterministic approaches (see for instance Syam and Côté [16]).

Sensitivity analysis has been the simplest approach used to address uncertainty [21], and it is simply applied by varying the values of the most uncertain parameters and evaluating whether those variations have impact or not on planning decisions. Nevertheless, in what concerns methods for planning the delivery of health care services in an uncertain environment, simulation has been the most used method (for example, Harper et al. [22]), since there is still little research considering stochastic approaches. However, some examples exist proposing stochastic models for health care planning. For instance, the multi-objective stochastic mathematical programming approach proposed by Cardoso et al. [23] to support the planning of a long-term care network, accounted for uncertainty in the demand and delivery of care. To describe the combinations of the considered uncertain parameters, a scenario tree with 81 scenarios was built.

### 2.2. Mental health planning

Regarding the case of the mental health sector, there is still little research on models considering its specificities. Moreover, most of the mathematical programming models have been developed to support the planning of mental health care services in the US context. An example is the model proposed by Leff et al. [4] for resource planning and policy evaluation to aid health planners making resource allocation decisions in the mental health care sector. Also, Franz et al. [3] suggested a chance-constrained goal programming approach (allows the transformation of a stochastic problem into an equivalent nonlinear deterministic model) to plan the delivery of mental health services. The objective function was defined to minimize deviations from the mental health system goals according to previously assigned priorities and/or weights.

Within this context, it is clear that there is need for developing research in this area applied to the

European context, in particular NHS-based systems. Since these models have already shown evidence to be successful for aiding the planning in other health sectors, they show great promise for being applied to plan the mental health care sector.

### 2.3. Estimates on future demand

So as to ensure an adequate planning of health care services, the previously reviewed methods should depart from information on current and future demand for care. Nevertheless, this type of information is often not available, and so there is need to develop methods to predict this. However, very little research has been done on this subject. When it comes to estimating and predicting future demand for health care services, in order to be integrated in mathematical programming models for health care planning, different approaches have been followed. Some studies have proposed methods to predict future demand for health based on information from current level of service utilization. Others predicted future demand based on the characteristics of the potential population in need for health care services.

In the mental health care planning literature, studies constructing future demand estimates in order to be integrated into mathematical programming models to support planning of mental health care networks were not identified. Within this context, along with the fact that the current supply of mental health care services is far from being enough to meet all the demand, it is essential to build estimates on future demand for mental health care.

## 3. Methodology

The methodology used in this study is illustrated in Figure 1. First, the future demand for mental health care was estimated (represented in green). Then, a mathematical programming model was developed to aid mental health planning (represented in blue).

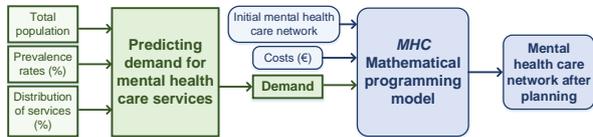


Figure 1: Representation of the methodology used in this study

### 3.1. Predicting demand for mental health care

To predict the future demand for mental health care: first, the prevalence rates are applied in order to determine the population with mental disorders; then, information on the distribution of services is used, resulting in the demand for each mental health care service (Figure 2).

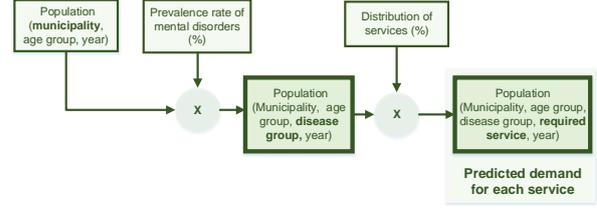


Figure 2: Representation of the methodology used to predict the demand for mental health care

The notation adopted to formulate the steps of the approach followed to predict the demand for mental health care services is presented in Table 1.

Table 1: Notation adopted to predict future demand for mental health care

Notation	Description
<b>Indices</b>	
$m$	Municipalities
$a$	Age groups
$d$	Mental disorders/Disease groups
$s$	Mental health care services
$y$	Years
<b>Parameters</b>	
$Pop_{may}$	Population from municipality $m$ and age group $a$ in year $y$
$Prev_d$	Prevalence of mental disorders $d$ (in %)
$Distrib_s$	Distribution of mental health care services $s$ (in %)
<b>Variables</b>	
$PMD_{mady}$	Population from municipality $m$ and age group $a$ with mental disorder $d$ in year $y$
$PMHC_{madsy}$	Population from municipality $m$ and age group $a$ with mental disorder $d$ requiring mental health care service $s$ in year $y$

To determine the population with mental disorders the multiplication of the population per municipality, age group and year ( $Pop_{may}$ ) by the prevalence rate of different mental disorders ( $Prev_d$ ) is computed (Eq. 1), resulting in the disaggregation of the data by disease group ( $PMD_{mady}$ ).

$$PMD_{mady} = Pop_{may} \times Prev_d \quad (1)$$

In order to find out how many people with the different mental disorders are in need of each mental health care service the population per municipality, age group, disease group and year ( $PMD_{mady}$ ) is multiplied by the distribution of mental health care services ( $Distrib_s$ ), as shown in Eq. 2, resulting in the disaggregation of the data by disease group ( $PMHC_{madsy}$ ).

$$PMHC_{madsy} = PMD_{mady} \times Distrib_s \quad (2)$$

### 3.2. Building the MHC model

This study proposes a mathematical programming model - the *MHC* model (with *MHC* standing for Mental Health Care) - to support the planning of a multi-service network of mental health care services in the context of NHS-based countries when multiple policy objectives need to be pursued. Figure 3 summarizes the key features of the *MHC* model, with special relevance given to the demand for mental health care services predicted in order to be integrated in the *MHC* model.

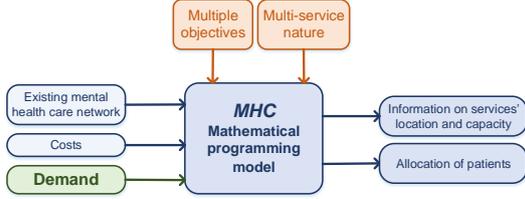


Figure 3: Representation of the key features of the *MHC* model

The *MHC* model supports mental health planning, both in terms of services location, capacity planning and allocation of patients to services. In particular, the model provides key information related to: (i) the opening and closure of services; (ii) the capacity (beds and human resources) needed in each service and location; (iii) the geographical distribution of the capacity across services and patient/disease groups; and (iv) the impact of the changes in the mental health care network on equity and cost-related objectives.

The notation used for the mathematical formulation of the model, organized into indices, parameters and variables is listed in Table 2. The model proposed in this study departs from the model proposed by Cardoso et al. [24], since several common characteristics can be found between the mental health care network and the long-term care network considered in this study. Nevertheless, several adaptations were made.

The model accounts for multiple equity objectives - equity of access (EA), geographical equity (GE), equity of service-specific utilization (ESU) and equity of disease-specific utilization (EDU) - pursued in any NHS-based system, as well as in the mental health care sector; and a cost objective, that is relevant in the current context of severe budget cuts and limited public health care spending. The objective of the model is thus to minimize costs and/or maximize several equity dimensions. These multiple objectives may be jointly considered depending on the planning circumstances, and thus the model can support mental health planning under different circumstances.

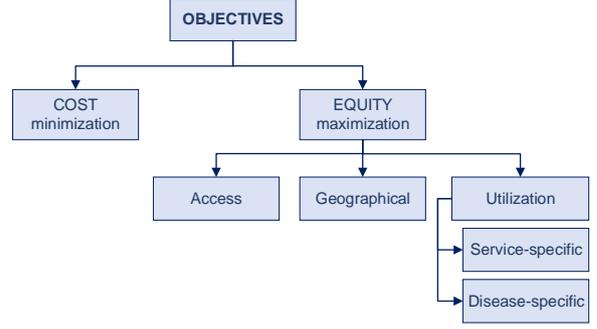


Figure 4: Objectives relevant in the mental health care sector

Table 2: List of parameters and variables

Notation	Description
<b>Indices</b>	
$t, t'$	Time periods
$d$	Demand points
$s, s'$	Mental health care services
$l, l'$	Location for services
$p$	Patient groups
$a$	Age groups
$h$	Human resources
$j, j'$	Type of mental health care provider
<b>Parameters</b>	
$ni_{dpat}$	Number of individuals from demand point $d$ , patient group $p$ and age group $a$ requiring service $s$ at $t$
$niD_{dt}$	Number of individuals from demand point $d$ requiring care at $t$
$niS_{st}$	Number of individuals requiring service $s$ at $t$
$niP_{pt}$	Number of individuals from patient group $p$ requiring care at $t$
$LOS_s$	Average length of stay, measured by the number of days in IC and RU service $s$ ( $s \in (S^1 \cup S^4) \subseteq S$ )
$\tau_t^{tot}$	Maximum total travel time at $t$ (in minutes)
$T^x$	Equity targets set by the DM at the end of the planning horizon (corresponding to the desired levels of achievement), $x = EA, GE, ESU, EDU$
<b>Variables</b>	
$ID_{dt}$	Number of individuals from demand point $d$ receiving care at $t$
$IS_{st}$	Number of individuals receiving service $s$ at $t$
$IP_{pt}$	Number of individuals from patient group $p$ receiving care at $t$
$TT_t$	Total travel time (in minutes) at $t$
$TO_t$	Total operational cost at $t$
$TI_t$	Total investment cost at $t$
$f^x$	Multiple objectives, $x = EA, GE, ESU, EDU, Cost$

### Defining the multiple objective function

In order to operationalize the objectives, the following measures are considered:

EA -  $Minf^{EA}$ : Minimization of the total travel time ( $TT_t$ ) for patients accessing mental health care

services throughout the planning horizon (Eq. 3). This objective ensures that patients receive the care they need as close as possible to their place of residence. The division of the total travel time ( $TT_t$ ) by the maximum total travel time ( $\tau_t^{tot}$ ) allows to represent this EA measure in a common  $[0,1]$  scale.

$$Minf^{EA} = \frac{\sum_{t \in T} TT_t}{\sum_{t \in T} \tau_t^{tot}} \quad (3)$$

GE -  $Minf^{GE}$  : Minimization of unmet need for the geographical area(s) with the highest level of unmet needs throughout the planning horizon (Eq. 4), avoiding a total lack of provision in some regions. GE is measured as the percentage of patients in the worst-off geographical area that do not receive care and is built to be represented in a common  $[0,1]$  scale. For example, a 0.4 value means that care is not provided to 40% of patients in need belonging to the geographical area with the lowest provision of mental health care.

$$Minf^{GE} = \max_d \left( 1 - \frac{\sum_{t \in T} ID_{dt}}{\sum_{t \in T} niD_{dt}} \right) \quad (4)$$

ESU -  $Minf^{ESU}$  : Minimization of unmet need for the service with the lowest level of provision throughout the planning horizon (Eq. 5), avoiding a total lack of provision of a particular service (usually the most expensive services or the ones with the highest LOS). ESU is represented in a common  $[0,1]$  scale. For instance, a 0.2 value corresponds to care not being provided to 20% of the patients in need for the service with the lowest level of provision. Hence, this objective ensures the maximum provision of the service with the lowest level of provision. Neglecting this objective can result in high variations in the utilization across different types of mental health care services.

$$Minf^{ESU} = \max_s \left( 1 - \frac{\sum_{t \in T} IS_{st}}{\sum_{t \in T} niS_{st}} \right) \quad (5)$$

EDU -  $Minf^{EDU}$  : Minimization of unmet need for the patient/disease group with the highest level of unmet need throughout the planning horizon (Eq. 6), preventing a total lack of provision for a particular patient/disease group. EDU is represented in a common  $[0,1]$  scale, with a 0.5 value meaning that care is not provided to 50% of patients belonging to the patient/disease group with the lowest level of provision.

$$Minf^{EDU} = \max_p \left( 1 - \frac{\sum_{t \in T} IP_{pt}}{\sum_{t \in T} niP_{pt}} \right) \quad (6)$$

According to the selected measures for GE, ESU and EDU objectives, presented above, higher levels of equity correspond to higher levels of unsatisfied demand (unmet need). For this reason, equity maximization is achieved through the minimization of its value.

Cost -  $Minf^C$  : Minimization of total cost (operational and investment costs) associated with the mental health care provision throughout the planning horizon (Eq. 7). Operational costs include costs associated to the operation of beds in IC and RU services and to the provision of AC, HBC and OU services. Investment costs include the investment in new beds, reallocation of beds between services in different locations and in the same location.

$$Minf^C = \sum_{t \in T} (TO_t + TI_t) \quad (7)$$

In case multiple objectives need to be considered in a given planning context, the proposed model allows dealing with these multiple objectives through the augmented  $\varepsilon$ -constraint method [18], applied by minimizing costs and imposing the equity-related objectives as constraints.

#### Defining the constraints of the model

The proposed model makes use of a set of constraints typically used in the literature:

- Assignment of patients' constraints - Patients receive the care they need in the closest available mental health service and cannot access services that are not within a maximum travel time;
- Opening and closure of services constraints - Opening/closing IC, RU and OU services is not allowed after deciding upon closing/opening it in a previous time period. Openings and closures are not considered for AC and HBC services, given that these services are partially provided within the scope of the primary health care network, already established in the context of an NHS-based country;
- Capacity constraints - Minimum and maximum number of beds (for IC and RU services) and patients assigned (AC, HBC and OU) per mental health service are imposed;
- Resources requirements constraints - Number of beds and human resources that should be made available per mental health services are calculated;
- Resources reallocation - The number of beds reallocated to service  $s$  from service  $s'$  should be equal to the number of beds removed from

service  $s'$  to service  $s$ ; and a maximum number of beds allowed to be reallocated from each IC and RU service is defined. Also, the reallocation of beds between services provided in different locations is only allowed in the first time period, while between services in the same location is always allowed. Furthermore, it is only possible to reallocate beds between services delivered by the same provider;

- Minimum service level - a minimum level of satisfied demand per mental health service should be guaranteed per time period.

#### 4. Case Study

The applicability of the *MHC* model is demonstrated through the resolution of a case study applied to the county level in the Great Lisbon region in Portugal over the 2016-2019 period. In particular, three planning contexts are defined representing scenarios with potential interest for real DMs in the mental health care sector in Portugal (Table 3).

Table 3: Planning questions and objective of the planning contexts under study. Legend: Max - maximize, Min - minimize

	Planning questions	Objective
A	How much would it cost to fully satisfy the demand?	Max EA
B	What is the lowest cost that ensures the achievement of all the equity targets?	Min Cost
C	What is the impact of the trade-off between cost and equity related objectives in the organization of the mental health care network?	Max EA, GE, ESU, EDU Min Cost

##### 4.1. Planning Context A

Under this planning context, it is assumed that the DM aims to plan the mental health network so as to ensure full demand provision. To achieve this, the model is run by imposing an additional constraint that ensures full demand satisfaction. In this situation, GE, ESU and EDU are already maximized. Within this setting, the EA is selected to be maximized.

Table 4 presents the number of new beds that are needed for IC and RU services over time, showing that a great investment in new beds is required in 2017. These results indicate that the current supply is far from being able to satisfy all the demand in this region, and so the bed capacity should increase significantly to fulfil the whole demand.

Table 4: Additional beds in which there is a need to invest over time under Planning Context A

	2017	2018	2019
IC	885	12	13
RU	29	1	3

The costs associated with the reorganization of the mental health care network are presented in Table 5. It can be seen that the highest investment takes place in 2017. This is an expected result since the investment is essentially related to the expansion in bed capacity necessary to meet the demand. Regarding the operational costs, they are approximately constant over time since the number of installed beds and the number of patients receiving care do not vary significantly. The cost of fulfilling the whole demand is approximately 822 M€ for the entire planning period. To sum up, the results show that the current supply is far from being enough to answer the mental health care demand in the Great Lisbon region.

Table 5: Operational and investment costs associated with mental health care provision over time under Planning Context A. Legend: Invest.-investment, Oper.-operational

Costs (M€)	2017	2018	2019	2017-2019
Invest.	19.08	0.26	0.32	19.67
Oper.	266.19	268.53	268.88	802.60
<b>Total</b>	<b>285.27</b>	<b>267.79</b>	<b>269.20</b>	<b>822.25</b>

##### 4.2. Planning Context B

This planning context is relevant when the planner wants to reorganize the mental health care network so as to ensure the attainment of equity targets define by the DM at the lower possible cost. To do so, the cost objective is selected and the model is run by imposing the achievement of target levels of equity as constraints.

Figure 5 depicts the evolution of the equity dimensions over the planning horizon. During the first two years, only the minimum level of demand has been satisfied since the model aims at minimizing costs. This explains why the GE, ESU and EDU levels remain constant in this period, with the value of 0.5. One can read that GE and ESU improve from 0.5 and 0.49 to 0.3 - attaining the respective targets ( $T^{GE}$  and  $T^{ESU}$ ), and EDU from 0.5 to 0.39, achieving a value below the defined target ( $T^{GE}$ ).

Regarding EA, its level is below the target during the entire planning period. As the other equities, the EA level is constant during the first two years.

However, by contrast, EA worsens from 0.16 to 0.22 (value in 2019), which corresponds to an increase in the travel time from 7 minutes to 9 minutes (per patient). This is due to the fact that during the first two years only 50% of the demand is being satisfied, whereas in the third year 70% of the demand is satisfied, as a consequence of achieving all the equity targets. Therefore, at the end of the planning horizon, the total travel time increases, and consequently the value of EA too.

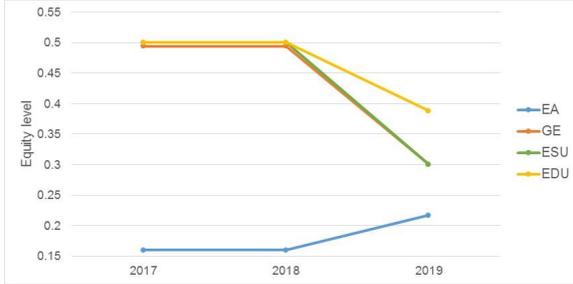


Figure 5: Evolution of equity levels over time under Planning Context B. Legend: EA - equity of access, GE - geographical equity, ESU - equity of service-specific utilization, EDU - equity of disease-specific utilization

Table 6 presents the minimum investment and operational costs that need to be incurred for reorganizing the mental health network in order to ensure the achievement of all the equity targets in 2019. In 2017, the investment cost is zero because the existing bed capacity is enough to satisfy the minimum level of demand. In 2018, there is a small investment due to an increase in demand caused by the population growth. Finally, in 2019, the investment is higher because a greater demand is being satisfied as a consequence of the achievement of the equity target levels. The operational costs are approximately constant in 2017 and 2018 and increase in 2019 for the same reason. Accordingly, results show that the highest total cost takes place in 2019.

Table 6: Operational and investment costs associated with mental health care provision over time under Planning Context B. Legend: Invest.-investment, Oper.-operational

Costs (M€)	2017	2018	2019	2017-2019
Invest.	0.00	0.17	1.86	2.03
Oper.	139.23	139.93	188.22	466.68
<b>Total</b>	<b>139.23</b>	<b>139.40</b>	<b>190.08</b>	<b>468.71</b>

According to the table, achieving all the equity

targets at 2019 requires a total cost of approximately 469 M€ for the entire planning period. This is 60% higher than the budget available for that period, which corresponds approximately to 285 M€. Therefore, the reorganization of the mental health care network associated with the attainment of the equity target levels defined by the DM is dependent on an increase of the available budget.

### 4.3. Planning Context C

In this planning context, it is explored how the current network of mental health care should evolve so that the multiple equity dimensions - EA, GE, ESU and EDU - are maximized and the cost is minimized. For this purpose, all the objectives are selected as objectives for the network planning, and the augmented  $\varepsilon$ -constraint method is used to explore the trade-off between them. However, the computational time needed to run the model with the 5 objectives is very high. For this reason, the model is run only with 3 objectives: cost minimization, maximization of EA and maximization of GE/ESU/EDU. The EA objective is included in all cases in order to ensure that patients receive the care they need as close as possible to their place of residence.

Figure 6 presents the Pareto Frontier obtained when running the *MHC* model with Cost, EA and GE objectives. All the solutions represented correspond to the (GE, Cost) solution with the minimum EA. Solution A represents the solution with the minimum total cost (418 M€) and the maximum level of GE (with the value 1), as measured by the level of unsatisfied demand in the geographical area with the lowest level of mental health care provision. The total cost of this solution is higher than the current budget available for mental health care provision in the Great Lisbon region for 2016-2019 period (285 M€). This result makes clear that an increase in the budget is needed for improving the delivery of mental health care in this region.

One one hand, as we move from solution A to E, the equity level decreases but the total cost remains constant. This means that it is possible to reorganize the mental health network so as to achieve lower levels of equity without increasing the bed capacity and thus incurring in more costs. Therefore, these solutions correspond to different configurations of the network across geographical areas. On the other hand, as we move from solution E to I, the total cost of reorganizing the network increases, as well the mental health provision in the worst geographical area. This means that when the GE is equal to 0.5 in the worst geographical area, in order to improve the GE level, it is necessary to expand the bed capacity by investing in new beds and consequently increasing the costs.

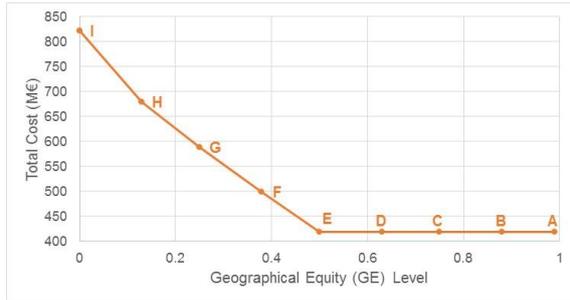


Figure 6: Pareto Frontier obtained when running the *MHC* model with Cost, EA and GE objectives

The maximum total cost (822 M€) and minimum equity level (with the value 0) is achieved under solution I, with the mental health network found under this solution allowing for full provision in all geographical areas, which corresponds to satisfying the whole demand (as observed in Planning Context A).

This type of analysis allows to explore how the improvements in one equity level compromises the other objectives - the same level of cost can be obtained with completely different network configurations. Accordingly, Pareto Frontiers are very useful to guide the planning, as DMs can select his/her most preferred solution.

#### 4.4. Sensitivity analysis

As an attempt to explore the impact of uncertainty on planning decisions, a sensitivity analysis to Planning Context B is employed - with this corresponding to the base case. Given the limitations of the data in use, a sensitivity analysis was carried out to evaluate the impact of changing key parameters on the model results. Specifically, sensitivity analysis was performed on the following parameters: (i) the demand for all types of mental health care services (IC, AC, HBC, RU and OU), because of the uncertainty associated with forecasts, which depend on the prevalence rates of mental disorders that have already a high level of uncertainty associated; and (ii) length of stay (LOS) of IC and RU services, because it is known that there are wide variations in the LOS among patients.

Results show that the total bed capacity that should be installed in the Great Lisbon region in 2019 is sensitive to changes in the demand and LOS. Consequently, the total cost also shows to be sensitive to changes in these parameters, with higher costs being incurred for satisfying higher levels of demand (as expected). Also, the locations for mental health care services are not robust to small changes in these parameters.

Furthermore, results show that the GE and ESU

levels are insensitive to changes in the demand and LOS, and this happens due to the binding constraints related to the achievement of the equity targets defined by the DM. However, the EA and EDU levels vary below the value of the respective equity target, with lower equity improvements (corresponding to higher equity levels) being achieved for higher levels of demand. Summing up, results show that the model outputs are sensitive to changes in the demand and LOS parameters, which indicates that their estimation should be done carefully.

## 5. Conclusions

The aim of this thesis is thus to develop a mathematical programming model - *MHC* model - to aid health planners in the management, designing and planning of networks of mental health care services, both at a strategical and tactical level, within the context of NHS-based countries. Estimates on future demand for mental health care are built based on the characteristics of the population with potential need of such care, and are integrated in the *MHC* model.

In summary, the MHC model was implemented in GAMS aiming to aid the planning of the entire range of mental health care services (IC, AC, HBC, RU and OU) while accounting for: a cost objective and multiple equity objectives - EA, GE, ESU and EDU. The *MHC* model by allowing the modelling of multiple objectives is a key tool to be used as to support planning under different circumstances - not only when a single objective is pursued, but also when multiple key policy objectives need to be addressed.

This thesis thus contributes to the literature by: (i) predicting future demand for mental health care services in order to be integrated in mathematical programming models for planning purposes; and (ii) proposing a mathematical programming approach to support planning decisions in the mental health care sector (a health sector not widely studied in the literature) within the context of NHS-based countries. The mathematical programming model not only contributes by considering the specificities of the mental health care sector, but also by: (i) considering the multi-service nature of mental health care; (ii) accounting for multiple policy objectives relevant in this sector; and (iii) exploring the impact of uncertainty on planning decision through sensitivity analysis.

The results obtained through the application of the *MHC* model to the Great Lisbon region show that the current mental health provision in this region is not adequate to meet its population needs, and the budget available for investment and operations is not enough to provide care with adequate levels of equities.

Running the model under these different planning contexts demonstrates that the model performs efficiently in computational terms, which supports its use to aid DMS with planning. Also, the sensitivity analysis performed shows that the model outputs are sensitive to changes in the demand and LOS parameters, which suggests that proper attention should be given to their estimation. Thus, the impact of demand and uncertainty in the planning of a mental health care network should be analysed with a stochastic approach. Nevertheless, taking into account the information retrieved from the application of the *MHC* model, the DM is able to make more informed decisions when planning mental health care networks.

At the end of this thesis, there are several developments that can be made in the future: (1) Build more accurate estimates on future demand for mental health care services based on the characteristics of the population with potential need for mental health care; (2) Extend the application of the *MHC* model to all the regions in Portugal and to the mental health sector within a NHS-based system in another country. Also, the spacial resolution of the model can be improved; (3) Ensure the applicability of the *MHC* model in real life context, in particular in the mental health care sector in Portugal and verify with DMs in the area whether new features considered relevant to the network planning can be added to the *MHC* model; (4) Apply the *MHC* model for different planning contexts. Also, the model can be applied for longer-term planning purposes; (5) Extend the *MHC* model to account for different types of human resources and include more types of mental disorders; (6) Explore alternative approaches to deal with the multiple objectives; (7) Develop a stochastic model to analyse the impact of demand and supply uncertainty in the planning of a mental health care network; and (9) Integrate the model with a Decision Support System, in order to interactively assist DMs.

All in all, this thesis addresses the gap that exists in the area of mental health care planning, given that few studies exist proposing methods to support planning decisions in this area.

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